Problem Chosen	2023 MCM/ICM Summary Sheet	Team Control Number
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Rotate the Olympics into a Better Future

Summary

Facing the situation that fewer and fewer countries are bidding for the Olympic Games, it is of far-reaching significance to establish an effective and sustainable new Olympic system. In support of the ICMG's work, we propose our recommendation: **Triple Olympic Circles (TOC)**. We address the following three questions: Is our pattern, TOC, better than the current pattern? How to implement TOC? And what benefits will TOC bring?

In Question 1, we establish the **Olympic Pattern Evaluation Model** (OPEM) to assess the costs and benefits to host the Olympic Games. A series of indicators are introduced to quantify multiple aspects of the Olympic Games. Accordingly, we establish 3 **primary indicators**, i.e., domestic pressure, degree of satisfaction and opportunities for future improvements. Each of them consists of 3 **secondary indicators**. Then we apply **EWM** and **AHP** to determine their weights. The results show that **TOC scores 1.31 times better than the current pattern,** proving the **feasibility** of our approach.

In Question 2, we use the **TOPSIS algorithm** to rate and select countries to join our TOC. In order to give developing countries equal chances to bid for the Olympics, we separate the current Summer Olympics into Summer (Aug 10th to Aug 27th) and Autumn (Oct 16th to Oct 30th) Olympics. After comprehensive scoring, Tokyo, Paris and New York are the top three cities suitable for Summer Olympics, while Jakarta, Sao Paulo and Peking are for Autumn Olympics.

For the Winter Olympics (Feb 5th to Feb 20th), we choose **K-Means and FCM algorithms**, introducing an **"ideal" city** for distance comparison. K-Means clustering sifts 12 candidates from 20 cities, after which the FCM selects Oslo, Stockholm and Innsbruck as nominees of the Winter Olympic Circle.

In Question 3, we pick Paris to evaluate the **effectiveness** of our system. The **Grey Forecast Model** is applied to predict the GDP of Paris from the year 2023 to 2025, followed by a **Multiple Linear Regression Model** that predicts the GDP growth rate after Paris joins the TOC. The results show that joining the TOC boosts Paris' economy, with a **GDP growth rate of 1.154% higher**.

Finally, based on the models and conclusions above, we add a **timeline** of our proposal and propose a complementary method: the **Olympic Community policy**.

Moreover, the advice to IOC is elaborated in detail in the memorandum.

Keywords: TOC, OPEM Model, EWM and AHP, TOPSIS, K-Means and FCM

Contents

1 Introduction

1.1 Problem Background and Restatement

Hosting the Olympics, the world's only truly global, multisport, celebratory athletics competition[1] , used to be a heated competition between candidate countries. The motivation to unite the world under the motto "Faster, Higher, Stronger, Together" is not only ignited by economic boost and infrastructure development, but also by the raise of national pride and the inspiration for future generations to pursue excellence in sports and other fields.

However, hosting such a large event certainly results in various short- and long-term negative impacts. Short-term effects include severe traffic congestion due to the influx of millions of visitors and disturbance of local people's life, and the long-term impacts involve possible deficit for the hosting country, displacement of local communities and environmental concerns such as construction-induced carbon dioxide emission. Therefore, countries are becoming more and more cautious about biding for the host of the Olympics, resulting in a decline in the number, as is shown in **Figure 1.1**.

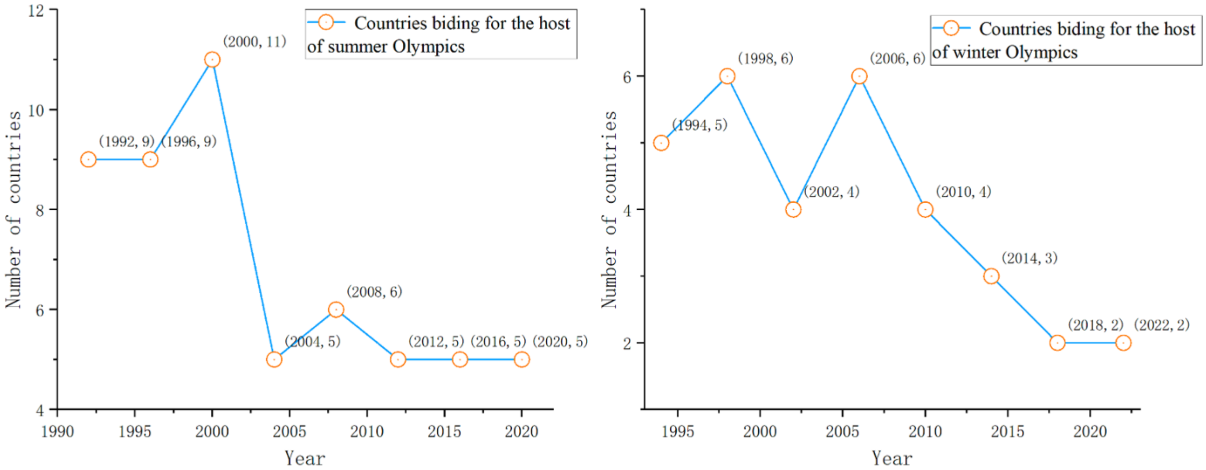

Figure1.1 The number of countries biding for the Summer and Winter Olympic Games

Challenged by ICMG[1], we recommend our approach – **Triple Olympic Circles**, to address the negative consequences aforementioned. Following our advice, the Olympics, as always, is ensured to continue to bring the world together through sports.

Considering the background information and the building metrics identified by ICMG, we are required to accomplish the following work:

- Provide a solution, ensuring the willingness and success of countries to host the Olympics.
- Evaluate the metrics, considering at least the 7 factors mentioned in the problem, creating a criterion to compare our proposal with the current Olympic pattern. Then prove the feasibility of our advice.
- Analyze the impact on the metrics, pointing out the strengths and weaknesses of our

[1] Interdisciplinary Committee on Modern Games.

approach based on the selected metrics.

- Outline the schedule, providing the timeline to implement our recommendation. Then write a memorandum describing our recommendation.

1.2 Our Work

Our work is sketched in the following flow chart, as is shown in **Figure 1.2**.

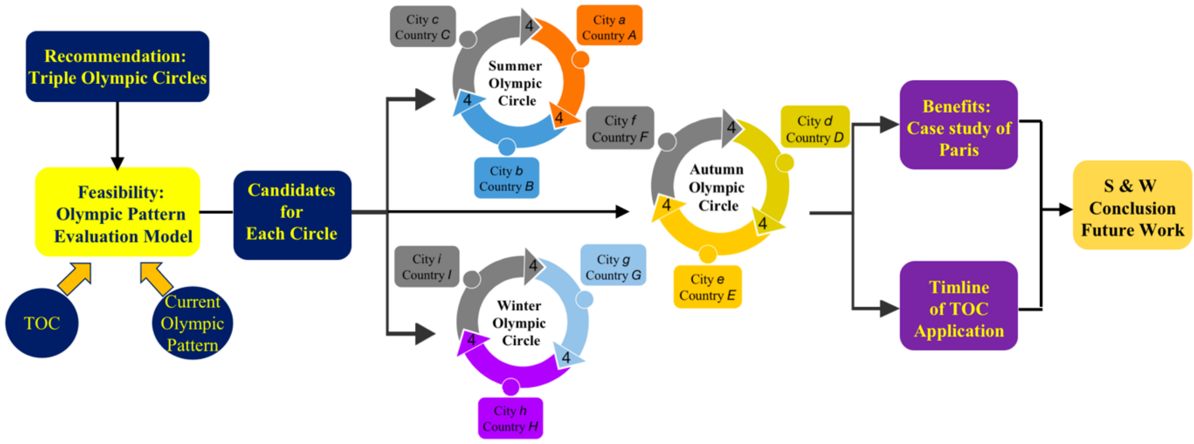

Figure 1.2 The flow chart of our work

Firstly, we establish our recommendation for the problem: **Triple Olympic Circles (TOC)**. We'll briefly introduce the model and its implementation in section 4.

Secondly, we use **Analytic Hierarchy Process (AHP)** and **Entropy Weight Measurement (EWM)** in series to obtain the fair coefficients from **three aspects** (**9 metrics**). We compare our approach against the current Olympic hosting pattern under these metrics, and the result, subsequently, yields the **feasibility** of our recommendation. We also discuss the impact of our strategy on these 9 metrics.

Thereafter, we apply **TOPSIS**, to select six countries (three countries each) from 25 candidates to host the Summer and Autumn Olympics in a cycle. We add special indicators to each Olympic Game for more accurate selection.

Subsequently, We use **K-Means** and **FCM** to cluster and pick three other countries (from 20 candidates) under another 2 special factors for the Winter Olympics.

Finally, we provide the IOC[2] with the **implementation timeline** of TOC, as well as the model evaluation, the description of future work, and a one-page memorandum.

2 Assumptions and Justifications

To simplify the problem, we make the following basic assumptions, each of which is properly justified.

➢ **Assumption 1: Land pressure can be expressed in terms of population density.**

[2] International Olympic Committee.

Justification: So far, almost all Olympic Games are held in the major cities of a country. All Olympic cities are at the top 10 of GDP in their countries. Some sparsely populated cities are not in the scope of our discussion, so we can express the land pressure in terms of population density.

- **Assumption 2: The indicators in our model are linearly independent of each other.**

  Justification: Based on the literature review[2], [3], we choose several indicators that manifests different faces of hosting grand sports events, and thus linearly independent. After combing these indicators to the building metrics in the problem, we construct our indicator system.

- **Assumption 3: The indicators are not disturbed by international incidents or policy.**

  Justification: Since the international environment is relatively stable, the indicators we select remains effective in short terms.

- **Assumption 4: The state of the Olympic system can be fully and scientifically reflected by limited and reasonably selected indicators.**

  Justification: In reality, factors that can affect the success of the Olympics are too plenty to be fully considered. Thus, this assumption is reasonable and helps avoid unnecessary dilemma when building the models.

- **Assumption 5: The budget to host the Olympic Games relies on the gross domestic product of the whole country.**

  Justification: It is impossible, from previous biding for the Olympics, for a particular city to host the Game on itself. It is always the case that the whole nation sponsors for the host city in order to build infrastructure such as sports venues and traffic systems.

3 Glossary and Symbols

The key mathematical notations used in this paper are listed in **Table 1**. And some variables that are not listed here will be discussed in each section.

Table 1: Notations used in this paper

Symbol	Description	Unit
c_{ij}	Normalized Dataset	\
EP	Economic Pressure	1
LP	Labor Pressure	Billion $
LU	Land Use (quantified as population density)	Person / km^2
M_{L}	The Average Scale of Large Stadiums	Ton
M_{M}	The Average Scale of Medium Stadiums	Ton
M_{S}	The Average Scale of Small Stadiums	Ton
AS_{T}	Athletes' Satisfaction Related to Local Temperature	℃
ASS_{traff}	Athletes' and Spectators' Satisfaction Related to Traffic Congestion	Hour h
STR_{envir}	Satisfactory Travel Related to Greenery Coverage	%
D_i^+	Euclidean Distance between the ith candidate to the "ideal" city	1(normalized)
D_i^-	Euclidean Distance between the ith candidate to the "hostile" city	1(normalized)

Other detailed notations, if necessary, will be mentioned and illustrated in each section.

4 General Description of TOC

Timetable and Event Arrangement for Athletes and Spectators

"The 2014 Olympics in Sochi, for example, ran up a bill of about $51 billion, while the 2008 Beijing Games cost nearly $40 billion." [4] According to Colleen Curry, the main reason for cities losing interest to host the Olympic Games is the strikingly high cost to build sports venues, especially large stadiums.

It is not cost-effective to separate Winter Olympics into two or three smaller Games. However, in order to relieve the burden of the hosting country, especially **possible developing countries**, it is practicable to divide the current Summer Olympics into two big parts: **the Summer Olympics** and **the Autumn Olympics**. As usual, Summer Olympics start on August 10th but last for only 18 days, while Autumn Olympics start on October 16 and last for 15 days. We allow the starting date to be advanced or delayed for less than 5 days.

In our new scenario, coined as **TOC**, athletes as well as spectators all over the world can enjoy three sports events every four years, as is shown in **Figure 4.1**. Specifically speaking, one Summer Olympic will be held in a city in country A, Paris in France, for instance; **One year later**, another city in country B will hold the Autumn Olympic. **In the winter of the third year**, a Winter Olympic is hosted in another country C from February 5th to February 20th (5 days adjustment permitted). Finally, **the fourth year** in the cycle will be **a gap year**, before another Summer Olympic is held.

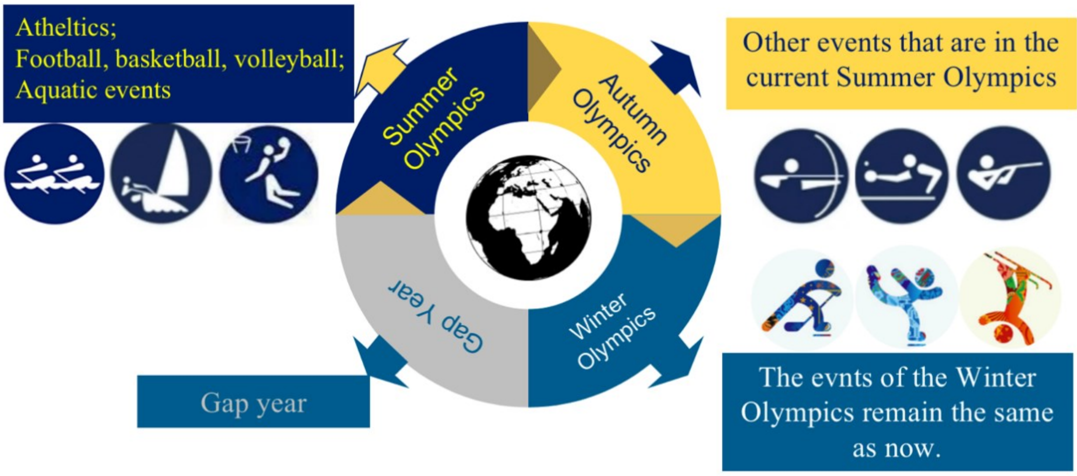

Figure 4.1 A four-year loop of Olympic events, from the perspective of athletes and spectators[3]

According to the construction costs of selected venues built for 2012 London Olympics[5] , stadiums and swimming pools cost approximately 1 billion dollars. With the aim of involving more countries to host the Autumn Olympics, we keep athletics, three major ball games and aquatic events still in the Summer Olympics. These events, though costly, engage the audience

[3] The logos are from the website: https://www.olympic.org/olympic-games-report

more than others. All other events currently host in summer are moved to the Autumn Olympics, as is shown in **Figure 4.2**.

Figure 4.2 The allocation of sports events in the Summer and Autumn Olympics

Fixed Host Countries

We will prove in section 5 that fixing the host countries (cities) in the TOC reduces the costs, and these countries enjoy greater benefits than in current scenarios. Thus, we decide to **choose three cities for each Olympic Circle**. These **9 cities in 9 different countries** form our Triple Olympic Circle.

5 The Olympic Pattern Evaluation Model

5.1 Data Normalization and Orthogonalization

The indicators can be categorized as follows:

⌘ Bigger and Better Indicators (BBI)

These metrics positively contribute to the final criterion of our Olympic Pattern Evaluation Model (OPEM).

⌘ Smaller and Better Indicators (SBI)

These metrics negatively contribute to the final criterion of OPEM.

⌘ Optimal Indicators (OI)

Indicators belonging to this category contribute to OPEM best at a certain value or in a certain range.

The aim of data processing is to **align these three categories into the first one** so as to compare our recommendation against the current Olympic pattern.

We denote the original data matrix as $X = \{x_{ij} | i, j = 1,2, \ldots, n\}$. After our data processing, the elements become x'_{ij}. For BBI such as the number of travelers, ASS_{traff}, STR_{envir}, $x'_{ij} = x_{ij}$. For SBI such as EP, LP, LU, etc. $x'_{ij} = \max(x_j) - x_{ij}$. For OI such as the local average temperature, we set the optimal point or region as b, then

$$x'_{ij} = 1 - \frac{|x_{ij} - b|}{\max(|x_{ij} - b|)} \tag{5.1}$$

The next step is data normalization, and we choose the **maximum-minimum normalization method**, i.e.,

$$c_{ij} \equiv \frac{x'_{ij} - \min(x'_j)}{\max(x'_j) - \min(x'_j)} \tag{5.2}$$

where c_{ij} is the normalized data.

5.2 Selection of Indicators

We choose **9 metrics to measure the success and the hosting sustainability of the Olympic Games**, namely 3 primary indicators (PI) with 3 secondary indicators (SI) each, as is shown in **Figure 5.1**.

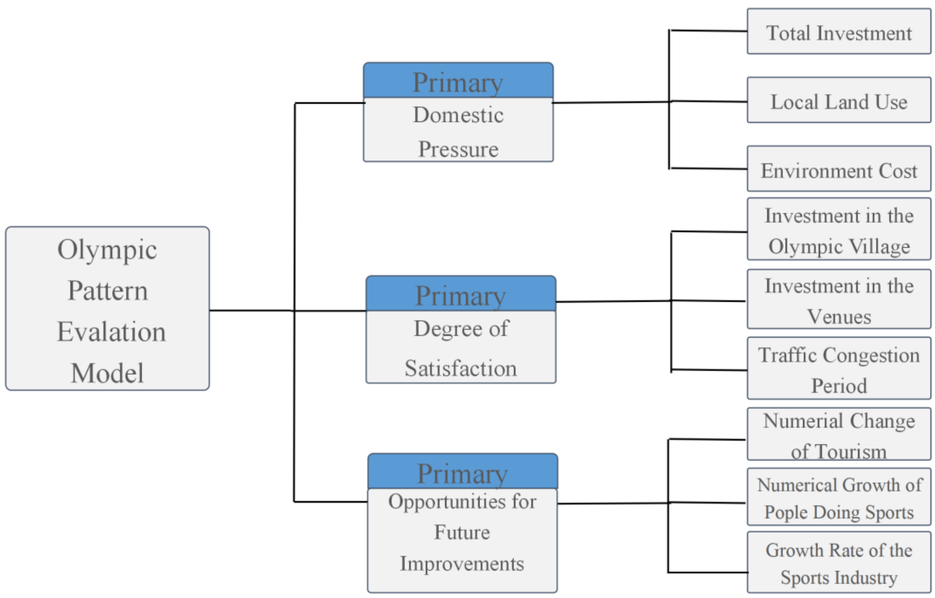

Figure 5.1 Three primary indicators each with three secondary indicators

- PI01 – Domestic Pressure

  The domestic pressure for the Olympic host includes the **total investment**, the **local land use** and the **environmental cost** measured by CO_2 emission.
- PI02 – Degree of Satisfaction

  For the second PI, the degree of satisfaction for both the athletes and spectators, we chose three SIs: the **investment in the Olympic Village**, the **investment in the venues** for athletes' training and competition, and the **traffic congestion period** to measure the convenience of traveling.
- PI03 – Opportunities for Future Improvements

  For the last PI, we choose the **numerical change of tourism** in the city in the year of the Olympic Games, the **numerical growth of people doing sports** and the **growth rate of the sports industry**.

5.3 AHP and EWM Analysis

5.3.1 EWM Determination of SI Weights

Based on our assumption of independence, we combine three PIs with 9 SIs as

$$W = \sum_{i=1}^{3} \alpha_i Y_i = \sum_{i=1}^{3}\sum_{j=1}^{3} \alpha_i (\beta_{ij} x_{ij}) \tag{5.3}$$

where Y_i stands for PI with weight α_i, and x_{ij} is the SI with weight β_{ij}.

EWM is applied to calculate β_{ij}. Information entropy judges the degree of dispersion of a certain indicator. The smaller entropy value is for x_{ij}, the greater its degree of dispersion, and thus the greater β_{ij} in the comprehensive evaluation.[6]

Specifically, we calculate the entropy value of the jth indicator x_{ij}:

$$e_{ij} \equiv -k \sum_{l=1}^{8} p_{lj} \ln p_{ij} = -\frac{1}{\ln 8} \sum_{l=1}^{8} \frac{c_{lj}}{\sum_{j=1}^{9} c_{lj}} \ln \frac{c_{ij}}{\sum_{j=1}^{9} c_{lj}} \tag{5.4}$$

where c_{lj} is the normalized data in formula (5.2) and the summation of l is based on data from 8 previous Olympic Games. Therefore, the information entropy redundancy $d \equiv 1 - e$ is within reach. Finally, we arrive at the weights of each indicator, i.e.,

$$\beta_{ij} = \frac{d_{ij}}{\sum_{j=1}^{9} d_{ij}} = \frac{1 + k\sum_{l=1}^{8} p_{lj} \ln p_{ij}}{\sum_{j=1}^{9}\left(1 + \sum_{l=1}^{8} p_{lj} \ln p_{ij}\right)} \tag{5.5}$$

5.3.2 AHP Calculation of PI weights

Although EWM gives us objective weights through entropy measurement, the current scenario that fewer and fewer countries bid to host the Olympics obliges us **to focus more on the domestic pressure** (investment, land use and environmental problems).

To handle the situation, we introduce AHP to adjust the weights. We think the degree of satisfaction and the opportunities for future improvements are of the same importance, while domestic pressure is a bit more important than them. Through the **Eigenvector Method**, we calculate the weights of PIs, with $\alpha_1 = 0.6$, $\alpha_2 = 0.2$ and $\alpha_3 = 0.2$ for domestic pressure, degree of satisfaction, and the opportunities for future improvements, respectively. Then, we allocate weights of secondary indicators according to their internal ratio and relative weights of PIs.

The consistency test for the PIs guarantees each α_i, as is shown in **Table 2**.

Table 2 Consistency test in AHP

Maximum Eigenvalue	Consistency Index (CI)	Random Index (CI)	Consistency Ratio (CR)	Result (CR<0.1?)
3	0.04	0.525	0.076	PASS

We can see that the consistency ratio is less than the threshold, hence, we are confident to determine β_j, as is shown in **Figure 5.2** below. It shows that the AHP algorithm has successfully adjusted the weights we find out with EWM, for the largest weights are all distributed to the first PI: domestic pressure.

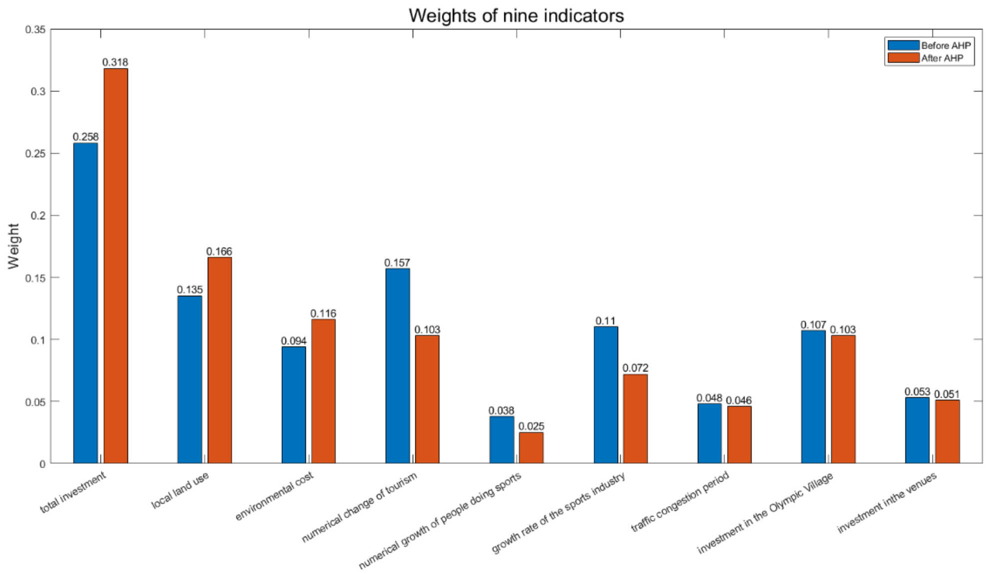

Figure 5.2 The weights of 9 SIs before AHP (blue bars) and after AHP (orange bars).

5.4 Comparison Between TOC and Current Olympic Pattern

In order to apply OPEM to our TOC recommendation, we select **Beijing, London and Rio de Janeiro** to be in our *summer cycle*; **Vancouver, Sochi and PyeongChang** as our candidates in the *winter circle*. Besides, we assume the Autumn Olympics will be held in the same place as the Summer Olympics.

Applying the SIs calculated in each of the cities mentioned above, we finally get the scores of our TOC and the current Olympic pattern, as is shown in **Figure 5.3**. Obviously, the TOC has a better performance, whose score is 1.31 times that of the current Olympic pattern.

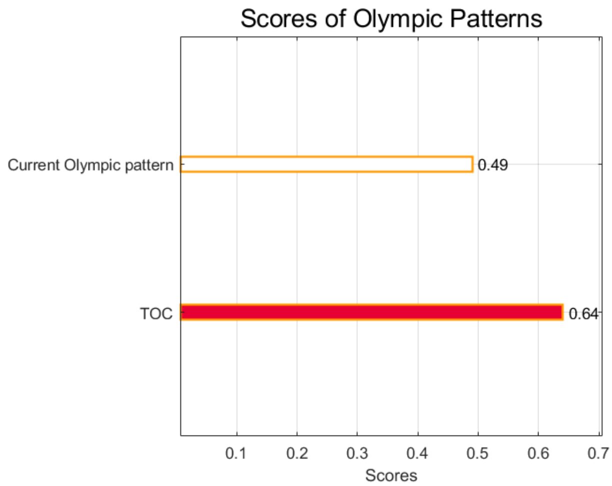

Figure 5.3 Comparison between the TOC and the current Olympic pattern

6 The First TOC: Summer Olympic Circle

There is no doubt that hosting the Summer Olympics will cost a lot of money, sometimes even exceeding the GDP of developing countries. We expect developed countries to be more suitable to host the Summer Olympics, for they are financially confident to build sophisticated large stadiums and, correspondingly, not only draw attention from spectators across the globe but also demonstrate their competitiveness in these high-profile projects.

On the basis of **Assumption 5**, we choose 25 cities in 25 countries as candidates for the Summer Olympics.

6.1 Model Establishment

In order to quantify whether a city is suitable for hosting the Summer Olympics a couple of times, we introduce six indices based on economic pressure and people's satisfaction.

6.1.1 Cost and Future Improvements: the First 3 Indices

- EP Economic Pressure

  This is an indicator of the capability for a given country to host the Summer Olympics. Though it is a particular city hosting the game, the whole country should provide financial support, so EP is defined as

  $$EP \equiv \frac{f_{\mathrm{RMS}}}{B} = \frac{C_{\mathrm{RM}} \cdot M_{\mathrm{tot,summ}}}{B} \tag{6.1}$$

  Where B is the **budget of sports industry** in a country, proportional to its Gross Domestic Product (GDP). **The cost of infrastructure construction** f_{RMS} is calculated by multiplying the cost of raw materials C_{RM} (converted into the cost of steel-reinforced concrete, USD / ton) and **the total amount of stadiums** for the Summer Olympics $M_{\mathrm{tot,summ}}$:

  $$f_{\mathrm{RMS}} \equiv C_{\mathrm{RM}} \cdot M_{\mathrm{tot,summ}} = C_{\mathrm{RM}} \cdot (1 \times M_{\mathrm{L}} + 3 \times M_{\mathrm{M}}) \tag{6.2}$$

  We estimate that 4 stadiums need to be built for a city to host the Summer Olympics (one large stadium M_{L} and three medium-sized stadiums M_{M}). Furthermore, in order to unify the parameters, we assume the following relationship among the scale of stadiums:

  $$M_{\mathrm{L}} = 3M_{\mathrm{M}}$$

  Hence, according to formula (6.1) and (6.2), we arrive at

  $$EP = \frac{1}{B} C_{\mathrm{RM}} \times \left(M_{\mathrm{L}} + 3\frac{M_{\mathrm{L}}}{3}\right) = 2\frac{C_{\mathrm{RM}} M_{\mathrm{L}}}{B} \tag{6.3}$$

- LP Labor Pressure

  Developed countries face more severe labor shortages than developing countries, and the average staff cost is higher. To quantify, we define

  $$LP \equiv AAS \times BT + SER \tag{6.4}$$

  where AAS is **average annual salary** and BT is the **time** required to build stadiums, proportional to $M_{\mathrm{tot,summ}}$. SER is the **service cost**.

- LU Land use

  The land use of the hosting city is characterized by its **population density** (person per km^2). The higher it is, the less spare land there is for the city to build Olympic infrastructure.

If a city happened to host Olympic Games in the recent thirty years, they do not have to build new stadiums but only concern about maintenance and refurbishment.

6.1.2 Satisfaction and Travel: Another 3 Indices

- AS_{T} Athletes' Satisfaction

  Athletes' satisfaction depends heavily on the local temperature T in summer. High temperature increases the possibility of sunstroke and heat exhaustion, while low temperature is hostile to water sports. Therefore, We set the suitable temperature range as $[18°\mathrm{C}, 23°\mathrm{C}]$. The interval orthogonalization of T yields AS_{T}, which means that the closer the average temperature is to the range, the higher score this city will get.

- ASS_{traff} Athletes' and Spectators' Satisfaction

  The athletes' and the spectators' satisfaction partly depend on the comprehensive service level supplied by the host city. We orthogonalize the average congestion time T_{conj} to describe the transportation capacity of each city, which represents ASS_{traff}. All the data about traffic jam comes from [7].

- STR_{envir}. Content Travel

  "Olympic traveling" has gone viral in recent years. Environment, therefore, plays an important role to raise people's satisfaction when they travel to a foreign country. We choose **greenery coverage** p_{GC} (%) to quantify the air quality, and its orthogonalization leads to STR_{envir}.

6.2 TOPSIS Algorithm

We define data matrix XS as the combination of six indices: EP, LP, LU, AS_{T}, ASS_{traff} and STR_{envir}. In the matrix, $XS_{ij} (1 \le i \le 25, 1 \le j \le 6)$ represents the jth index of the ith city. After normalizing the data with method mentioned in OPEM, we have the normalized matrix ZS, which is also a 25×6 matrix.

6.2.1 Determination of an "Ideal" City

After normalization and orthogonalization, we calculate the optimal solution and the worst solution. We take **the largest numbers in each index column** to form the ideal solution vector, i.e., **an "ideal" city**:

$$Z_j^+ = \max\{Z_{1j}, Z_{2j}, Z_{3j}, \dots, Z_{25j}\} \tag{6.5}$$

with $Z^+ = [Z_1^+, Z_2^+, Z_3^+, \dots, Z_{25}^+]$. Similarly,

$$Z_j^- = \min\{Z_{1j}, Z_{2j}, Z_{3j}, \dots, Z_{25j}\} \tag{6.6}$$

yields the worst solution vector, with $Z^- = [Z_1^-, Z_2^-, Z_3^-, \dots, Z_{25}^-]$.

All other candidates are compared to these two cities and are scored according to the clustering. The aim of this approach is to determine which cluster is our final choice.

6.2.2 Quantify the criterion for candidate selection

Define Z_i as the indices of the ith city, and the rating for each scenario can be calculated by

$$\frac{Z_i - Z^-}{Z^+ - Z^-} = \frac{Z_i - Z^-}{(Z^+ - Z_i) + (Z_i - Z^-)} \tag{6.7}$$

The distance between a candidate and the "ideal" city $D_i^+ = Z^+ - Z_i$, or the "hostile" city $D_i^- = Z_i - Z^-$ are:

$$D_i^+ = \left(\sum_{j=1}^{6} w_j\left(Z_j^+ - Z_{ij}\right)^2\right)^{\frac{1}{2}}, D_i^- = \left(\sum_{j=1}^{6} w_j\left(Z_j^- - Z_{ij}\right)^2\right)^{\frac{1}{2}} \tag{6.8}$$

where w_j **is the weight for each index**.

Next, we measure the relative distance:

$$C_i = \frac{D_i^-}{D_i^+ - D_i^-} \tag{6.9}$$

Formula (6.7) and (6.8) ensure that C_i ranges from zero to unity. The better a city is capable of hosting an Olympic game, the higher the value of C_i. Since some indices play more important roles while some exaggerate the real difference, we use AHP again to adjust the weight of each index.

6.3 Results of the Summer Olympic Circle

Plotting with MATLAB, we get the score of each city, as is shown in **Figure 6.1**.

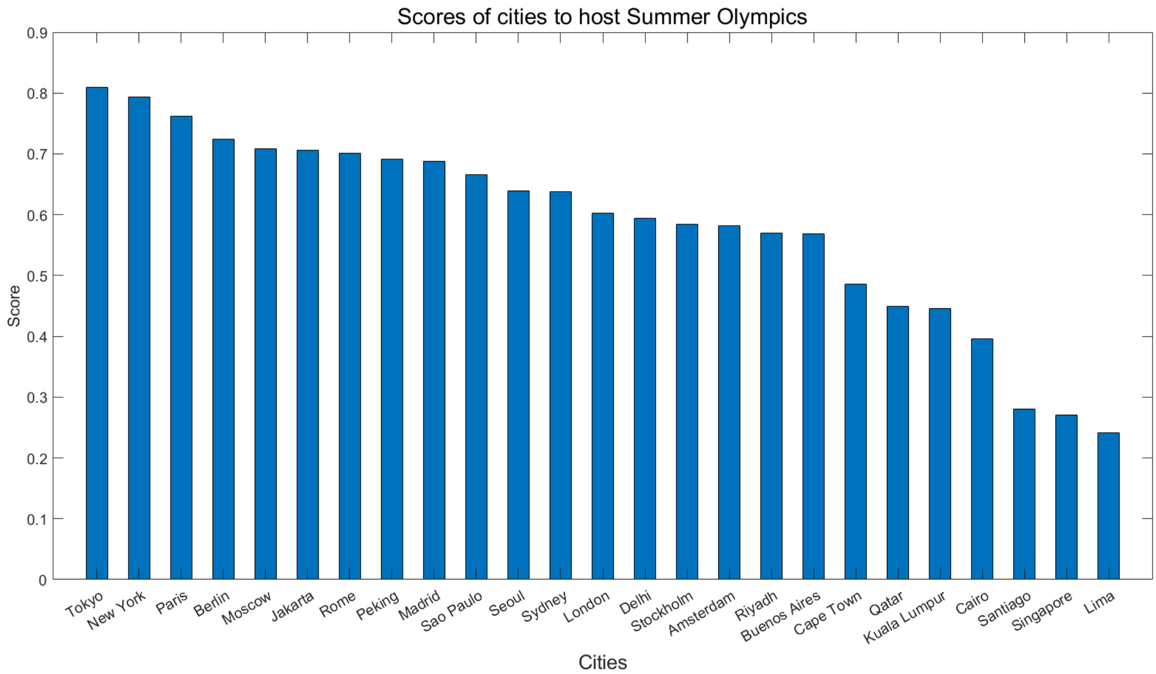

Figure 6.1 Scores of cities to host the Summer Olympics

According to the outcome, developed countries have a greater advantage compared with developing countries. This result is congruous with our expectations mentioned before. **Tokyo ranks first, with New York and Paris second and third place**. The scores of most candidates are over 0.5, since developing countries chosen by us are usually regarded as potentially competent. However, for those countries whose land area and economic power are not large enough, it is difficult to host such a big sports event.

Our suggestion is that **the top three cities take turns to hosting the Summer Olympics**. Especially, as Tokyo has already hosted the 2020 Olympics, **it can be the beginner of the summer TOC after the policy takes effect**. Paris also has an advantage, for it is going to host the 2024 Summer Olympics, so **New York has to build new venues as soon as it agrees to be a member of the Summer TOC**.

7 The Second TOC: Autumn Olympic Circle

Developing countries need an opportunity to show national strength and enhance their international prestige. Hosting international sports events provide just that. For instance, China has hosted a Summer and a Winter Olympic Game, Brazil has hosted a World Cup and a Summer Olympic Game, and Qatar and Saudi Arabia are both interested in hosting the World Cup.

However, for many developing countries, it is a daunting task to host a traditional Olympic game, due to the lack of financial capability, infrastructure, advanced traffic system, etc. The Autumn Olympics meet exactly their need and are within their reach. According to our advice, there is no need to build large stadiums, and only **three quarters of the budget of a traditional Olympic is enough**. What's more, the opening ceremony can also be simpler than the Summer Olympics and therefore more cost-friendly.

7.1 Model Tuning

To host an Autumn Olympic Game, some variables should be adjusted compared with the Summer Olympics.

We add an **attenuation factor $\boldsymbol{\mu}$** to simulate countries that refuse to allocate the same sum of budget as Summer Olympics. The higher a country's level of sports development and comprehensive national strength, the smaller μ is. So, B should be replaced by $B' \equiv \mu B$.

Hence, the economic pressure EP is recalculated as:

$$EP = \frac{C_{\text{RM}} \cdot M_{\text{tot,aut}}}{B'} \tag{7.1}$$

Where the total amount of stadiums $M_{\text{tot,aut}} \equiv 1 \times M_{\text{M}} + 4 \times M_{\text{S}}$.

Temperature does not involve as important as in the Summer Olympics, for all the cities are in an agreeable temperature if we stipulate that the Autumn Olympics should be in October.

However, we have to consider the **attitude difference between developed and developing countries** towards the Autumn Olympic Games. It is not only reflected in budget allocation, but also in strategic planning in policy guidelines and the enthusiasm for bidding for the Autumn Olympic Games. We introduce an **attitude factor $\boldsymbol{\alpha}$**, which is the reverse of μ by orthogonalization, to describe a country's attitude.

We have to emphasize that α is usually correspondent to GDP, but not always. Different countries have different policies and we need to adapt α for some countries like Qatar, Saudi Arabia, India and China.

Till now we can define the 25×6 data matrix XA as:

$$XA = [EP, LP, LU, STR_{\text{envir}}, ASS_{\text{traff}}, \alpha] \quad (7.2)$$

And we are ready to evaluate each candidate to host the Autumn Olympics.

7.2 Results of the Autumn Olympic Circle

Using TOPSIS, we can get the results of the second TOC, as is shown in **Table 3**.

Table 3 The ranking and selection of cities hosting the Autumn Olympics

City	Score	Rank
Jakarta	0.732103	1
Sao Paulo	0.723746	2
Peking	0.704354	3
Buenos Aires	0.670557	4
Riyadh	0.659797	5
Doha	0.644808	6
Tokyo	0.632950	7
Cape Town	0.609541	8
New York	0.596030	9
Paris	0.583514	10
Berlin	0.576096	11
Sydney	0.564517	12
Kuala Lumpur	0.561251	13
London	0.542707	14
Rome	0.529190	15
Delhi	0.526246	16
Moscow	0.518360	17
Cairo	0.514323	18
Santiago	0.510759	19
Seoul	0.492782	20
Madrid	0.484011	21
Amsterdam	0.402249	22
Lima	0.395965	23
Stockholm	0.377820	24
Singapore	0.182821	25

The top 6 cities are all in developing countries. Therefore, we are confident to say that our approach encourages more developing countries to host the Olympics.

Jakarta in Indonesia ranks 1st because of its powerful economic tenacity and a strong willingness to host the Game. **Sao Paulo and Peking rank 2nd and 3rd respectively**. Besides, Qatar and Saudi Arabia are both in the top 6. Their strong desire to host compensates for the lack of total GDP. We can find proof in reality that they have successfully hosted World Cup and Saudi Grand Prix.

In order to clarify **the influence of strong willingness to raise their international profile**, we compare the ratings of cities with their GDP, as is shown in **Figure 7.1**.

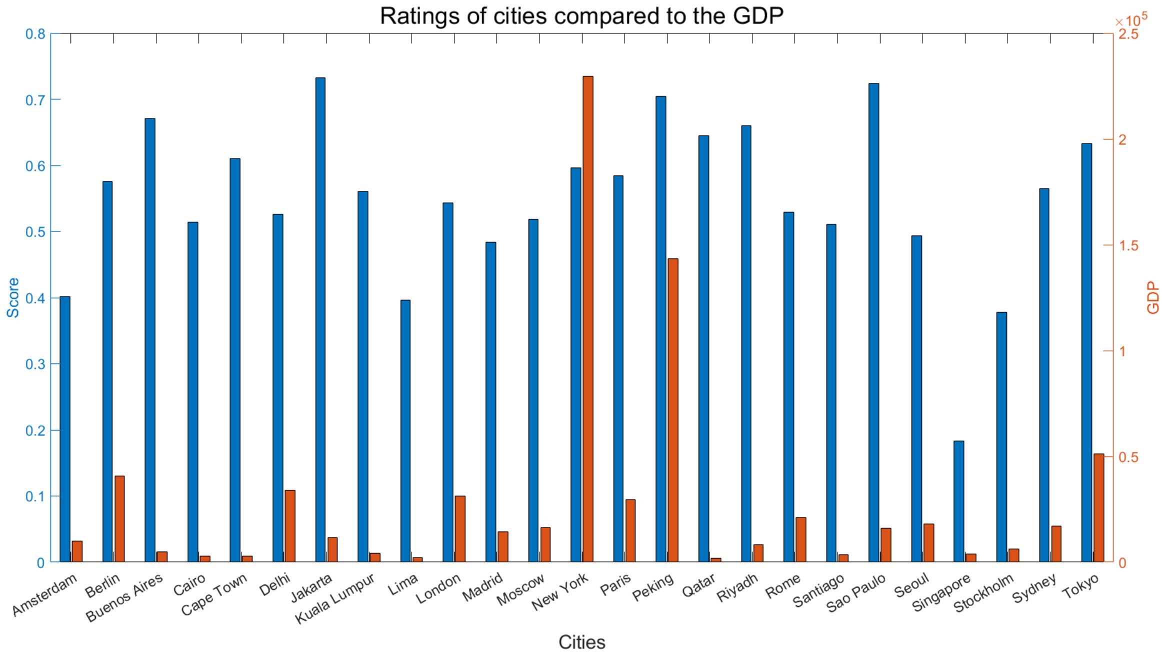

Figure 7.1 Ratings of cities to host the Autumn Olympics compared to the GDP

We can see from **Figure 7.1** that when choosing cities to host the Autumn Olympics, GDP is no longer a key constraint. Almost all the cities on our shortlist have chances to compete for the sponsorship qualification.

We suggest **IOC negotiate with the top three countries in the score ranking first**. Buenos Aires in Argentina, Riyadh in Saudi Arabia and Doha in Qatar also have a high score and they can be the backup team of the Autumn Olympic Circle.

8 The Third TOC: Winter Olympic Circle

8.1 Selection of Indicators with 2 Special Winter Factors

In contrast to the first and the second TOC, the conditions vary considerably for the third TOC. Therefore, we adjust the metrics in the fourth section and introduce **2 new factors** for the optimal choice:

- AS_{TW} Athletes' Satisfaction

  Athletes' satisfaction depends heavily on the local temperature T in winter. We set the suitable temperature range as $[-2℃, 5℃]$. The interval orthogonalization of T yields AS_{TW}. Considering the possibility of inappropriate conditions such as polar night, **the latitude** $\mathcal{L}$ is involved as a complementary indicator.

- ξ Mountainous Degree

  Various games in the Winter Olympics involve mountains, such as alpine skiing, freestyle skiing and biathlon. Therefore, whether or not a candidate location is harnessed with **a considerable amount of mountainous area** becomes crucial for hosting the Winter Olympic Games. We credit each candidate city with ξ =1, 0.5 or 0, based on the data from [8] . Besides, ξ serves as **the degree of satisfaction of athletes**, since the more eligible the land is for the competitors, the more contented these athletes would be with the Game.

LU and STR_{envir} are the same as the 6th section. However, the economic pressure EP is adjusted to

$$EP \equiv \frac{f''_{\text{RMS}}}{B''} \tag{8.1}$$

where the cost of infrastructure construction f''_{RMS} and the total amount of stadiums M''_{tot} are now defined as

$$f''_{\text{RMS}} \equiv C_{\text{RM}} \cdot M''_{\text{tot}} = C_{\text{RM}} \cdot (1 \times M_{\text{L}} + 2 \times M_{\text{M}} + 2 \times M_{\text{S}}) \tag{8.2}$$

That is, 1 large stadium, 2 medium-sized stadiums and 2 small stadiums are required to host the Winter Olympic Games. Besides, the scaling relation goes like:

$$M_{\text{L}} = 3M_{\text{M}} = 5M_{\text{S}}$$

Hence,

$$EP = \frac{C_{\text{RM}}}{B''} \cdot \left(M_{\text{L}} + 2\frac{M_{\text{L}}}{3} + 2\frac{M_{\text{L}}}{5}\right) = \frac{31}{15}\frac{C_{\text{RM}}M_{\text{L}}}{B''} \tag{8.3}$$

Accordingly, the labor pressure LP becomes

$$LP = AAS \times BT'' + SER \tag{8.4}$$

8.2 Fuzzy C-Means Determination of Hosting Cities

Based on the six indicators discussed in subsection 6.1, we use the **Fuzzy C-Means clustering method (FCM)**, an improved K-Means clustering, to determine three cities that shall be our candidates to host the Winter Olympic Games.

8.2.1 Data Collection and Pre-processing

Before applying our method, we **select 20 cities across the globe**, from countries across the globe. Our selection is based on a preliminary judgement according to the six indicators. In this way, we build our dataset.

We standardize the dataset to ensure that each variable contributes equally to the clustering process, and that the clustering is based on the relative distances between data points rather than the absolute values of the variables

Once the standardization is completed, the orthogonalization technique is applied to align all 6 evaluation metrics.

8.2.2 Selecting 12 Cities after K-means Clustering

In K-means clustering, n elements of the dataset, typically viewed as n vectors, are partitioned into k disjoint subsets. Any vector $\vec{v}$ in subset S_i is closer to the centroid of S_i than it is to any $S_j \neq S_i$, where $i,j \in [1,k]$ [9]. Euclidean distance $d_{\alpha\beta} \equiv |\overrightarrow{v_\alpha} - \overrightarrow{v_\beta}|, \alpha,\beta \in [1,n]$ is typically used as the measurement of closeness. The cluster centers are then updated as the mean of all data points are assigned to that cluster. Iterate $\vec{v}$ assignment and update each centroid until the cluster assignments no longer change and the clustering is accomplished.

Therefore, the "ideal" city could be represented as a new vector $\overrightarrow{v^*}$ and should be clustered, in the credible outcome, only with itself, since it serves as a reference to other cities.

Matlab calculation yields the clustering of 30 candidates and the "ideal" city $\overrightarrow{v^*}$, as is shown in **Figure 8.1**.

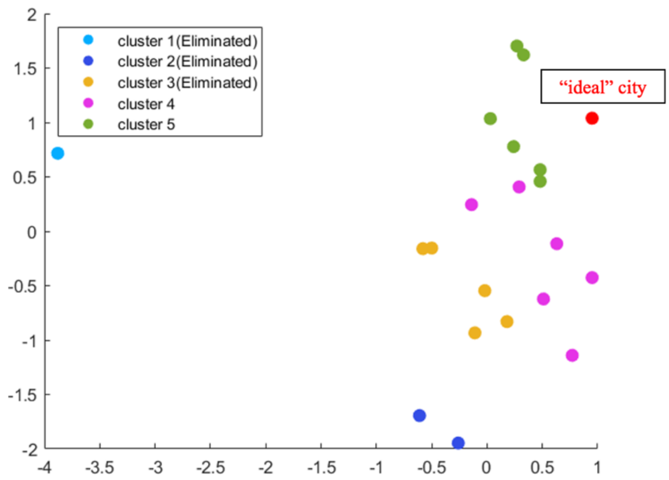

Figure 8.1 K-Means clustering for 12 candidates form 20 cities

According to the clustering, three clusters that are **farthest from the "ideal" city** need to be eliminated from our further evaluation. The eight cities that are sifted out based on our approach are: Lausanne (Swiss), Reykjavik (Iceland), Valle Navedo (Chile), Auckland (New Zealand), Montevideo (Uruguay), Sydney (Australia), Victoria (Seychelles) and Seoul (Korea).

From the discussion above, we choose 12 cities for further exploration.

8.3 FCM Clustering and the Final Designation

K-Means algorithm is applied to generate a basic sift of cities. However, the division of data into each cluster is rather hard and sometimes infeasible unless the number of S_i is so large that clustering itself is meaningless. Due to its intrinsic weakness, we apply FCM for more accurate analysis. FCM allows data points to belong to more than one cluster with varying degrees of membership (flexible "fuzzy" division) [4].

8.3.1 Introduction of FCM Algorithm

The aim of FCM clustering is to maximize the similarity of data within one cluster while minimizing data similarity between different clusters [11]. Compared to the K-Means method used in section 8.2, it harnesses the following privileges:

- More flexible in data clustering;
- Capable of capturing the fuzzy boundaries between clusters;
- More robust to noise and outliers;
- Capable of handling datasets with overlapping clusters

The steps of FCM clustering are sketched in the flow chart (**Figure 8.2**):

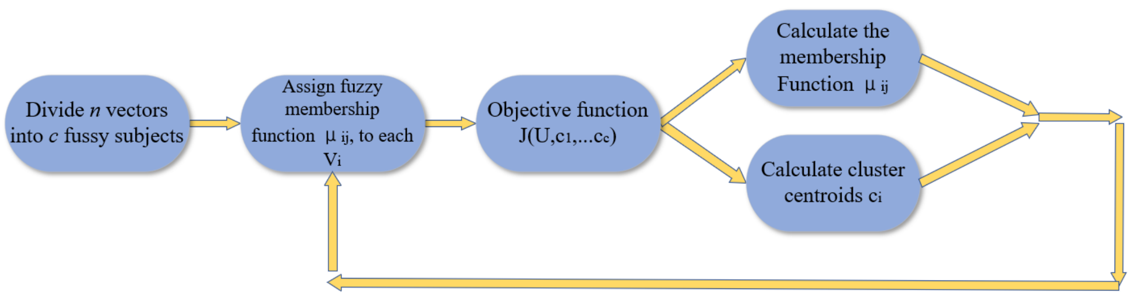

Figure 8.2 The four steps of FCM clustering

FCM firstly divides n vectors into c fussy subsets C_j. Our selected 12 cities are represented by $\vec{v_i}$ $(i = 1,2, \dots ,12)$, and $\overrightarrow{v_{cj}}$ stands for the centroid of each fuzzy group $(j = 1,2,, \dots, c)$. Then, the degree of membership of a data point in a group is introduced and is represented by a fuzzy membership function $\mu_{ij} \equiv \mu(\vec{v}_i; \overrightarrow{v_{cj}}) \in \{U|0 \leq \mu_{ij} \leq 1\}$. The degree of membership indicates the degree of similarity between the data point $\vec{v}_i$ and the cluster centroid $\overrightarrow{v_{ci}}$. Hence,

$$C_j = \{(\mu(\vec{v_i}, \overrightarrow{v_{cj}}), \vec{v_i}) | \vec{v_i} \in V\} = \{(\mu_{ij}, \vec{v_i}) | \vec{v_i} \in V\} \tag{8.5}$$

where V is the total dataset with data from 12 cities processed in section 8.2. The sum of the membership values for each data point is equal to 1, i.e.,

$$\sum_{i=1}^{n} \mu_{ij} = 1, \forall j = 1,2, \dots, c \tag{8.6}$$

The objective function J in FCM serves to minimize the sum of squared errors between $\vec{v_i}$ and $\overrightarrow{v_{cj}}$, weighted by the fuzzy membership values μ_{ij}. J takes the form of

$$J(U, c_1, \dots, c_c) = \sum_{j=1}^{c} J_j = \sum_{j=1}^{c} \sum_{i=1}^{n} \mu_{ij}^m d_{ij}^2 \tag{8.7}$$

where $c_1, \dots, c_c$ are on behalf of cluster centroids in one fuzzy group. $\boldsymbol{m} \geq \mathbf{1}$ **is the fuzzy exponent**, and d_{ij} is the distance defined in subsection 8.2.3.

Due to the constraints in formula (8.6), the objective function should be complemented with n Lagrange Multipliers λ_i:

$$\begin{aligned}\bar{J}(U, c_1, \dots, c_c; \lambda_1, \dots, \lambda_n) &= J(U, c_1, \dots, c_c) + \sum_{i=1}^{n} \lambda_i \left(\sum_{j=1}^{c} \mu_{ij} - 1 \right) \\ &= \sum_{j=1}^{c} \sum_{i=1}^{n} \mu_{ij}^m d_{ij}^2 + \sum_{i=1}^{n} \lambda_i \left(\sum_{j=1}^{c} \mu_{ij} - 1 \right)\end{aligned} \tag{8.8}$$

Calculate the minimum of formula (8.3), and we arrive at the cluster centroids:

$$c_i = \frac{\sum_{i=1}^{n} \mu_{ij}^m v_i}{\sum_{i=1}^{n} \mu_{ij}^m} \tag{8.9}$$

And the membership function under formula (8.6) is:

$$\mu_{ij} = \frac{1}{\sum_{k=1}^{c} \left(\frac{d_{ij}}{d_{kj}} \right)^{\frac{2}{m-1}}} \tag{8.10}$$

Thus, we are ready to update the membership function and itinerate these steps until J is lower than a given threshold.

8.3.2 The Optimal Choice

Applying $n = 12$ to the FCM algorithm, we pick up 3 hosts from 12 candidates selected by K-Means. The clustering and the corresponding "ideal" city are shown in **Figure 8.3**.

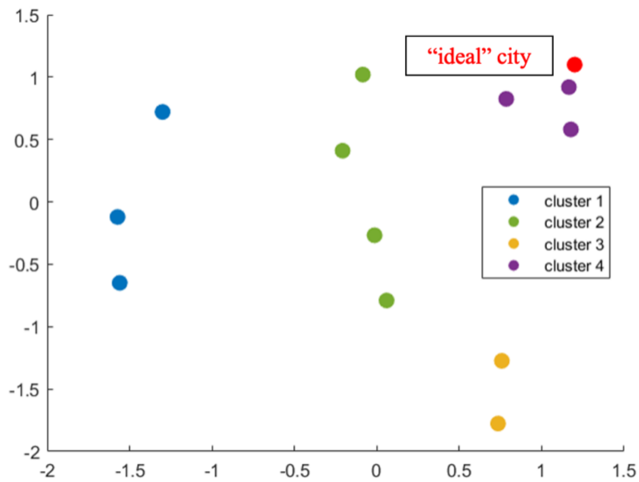

Figure 8.3 The optimal choice of three cities hosting the Winter Olympic Game

From the FCM analysis, we choose CLUSTER 4, which is **the nearest to the "ideal" city**, as our optimal conclusion. The three candidates in this cluster are: **Oslo, Stockholm and Innsbruck**. Oslo stands out mainly due to the alpine landscape, while Stockholm enjoys high greenery coverage and relatively low labor pressure. Innsbruck makes it way to the *winter circle* because of its well-developed infrastructure and perfect winter temperature. It is worth mentioning that all three cities enjoy high scores on Mountain Degree and Athletes Satisfaction.

Our suggestion is that **Oslo, Stockholm and Innsbruck take turns hosting the Winter Olympics**. Especially, as Innsbruck has already hosted 2 Olympics before, **it can be the beginner of the winter TOC after the policy takes effect**.

9 Model Evaluation and Timeline Proposal

9.1 Benefits of joining the TOC: Case Study of Paris

Based on the GDP values of Paris from 2006 to 2022, we apply a **grey forecast model** to make projections for Paris' GDP for the next three years **without joining the TOC**. The GDP in year 2023($t = 1$) to year 2025($t = 3$) is calculated from

$$\mathrm{GDP}(t) = \left(\mathrm{GDP}(\mathrm{t}_0) - \frac{b}{a}\right) e^{-a(t-1)} + \frac{b}{a} \tag{9.1}$$

where $a = -0.016, b = 4921.57$ is measured from linear regression [10] . The accuracy of our prediction is verified by the posterior difference test method.

In order to consider **the impact on GDP after Paris joins the TOC**, **we construct a multiple linear regression model** to predict the average GDP growth rate for the next three years. We selected impact indicators such as the percentage of **young adults**, percentage of **high-tech industries**, total exports, employment rate, etc. After **the F-test** of the model, we obtain the prediction as in **Figure 9.1**. The GDP of Paris will rise faster after becoming a member of the TOC. Compared with natural growth, the annual GDP growth is **1.154% higher**, which indicates the positive effect of the TOC on the host country.

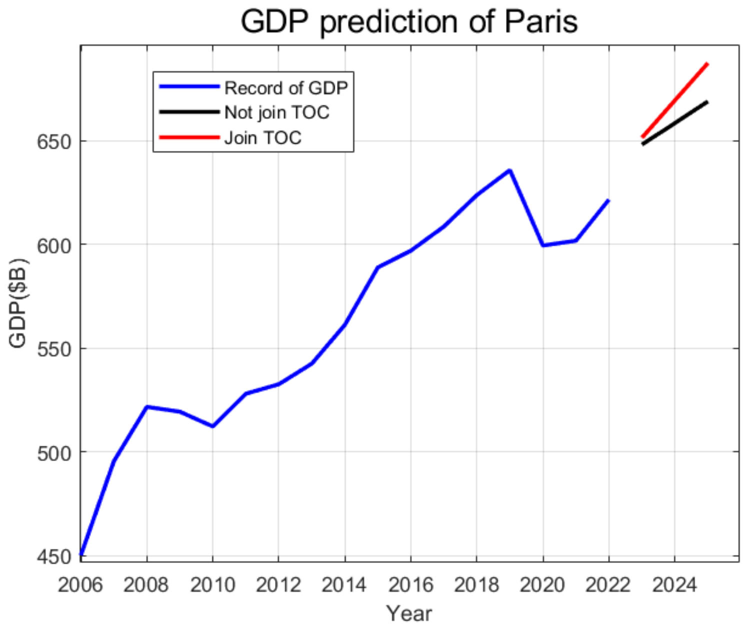

Figure 9.1 Benefit for joining the TOC, manifested by GDP growth.

9.2 Strengths and Weaknesses

9.2.1 Strengths

- Lower costs and long-term benefits

Our solution will bring a periodical tourism boom and economic benefits to the countries in the TOC, and the cost is lower than the current Olympic pattern in the long run. Without the necessity of significant investment, more countries will be promoted to bid for the Olympic Games.

- Higher degree of participation

  The Autumn Olympic Games mentioned in our solution require fewer expensive venues, and the economic pressure will be correspondingly lower. We believe the presentation of the Autumn Olympics will increase the participation of developing countries in the Olympic Games, and make the spirit of Olympics spread worldwide.

- Flexible time

  The GAP YEAR in our TOC is deliberately set to leave room for the IOC and the host when facing some international conflicts. If not, when a global pandemic erupts, there will be time conflicts, resulting in a lack of preparation. What's more, we try not to let the Olympics meet World Cup. This measure takes the practicality of the TOC into account.

9.2.2 Weaknesses

- Unexpected events

  The assumptions of our model do not take into account the impact of unexpected events such as epidemics and other large-scale emergencies around the world.

- Subjectivity of clustering method

  The number of clusters, the initial centroids of each cluster, as well as the fuzzy exponent in FCM, are subjectively chosen, which brings some extent of uncertainty to the model. We try to minimize subjectivity by running our model several times, changing the number of clusters and the initial positions of centroids.

9.3 Timeline of TOC Implementation

According to the Olympic Charter, hosts generally should be elected seven years in advance to prepare for the Olympic Games. Since the 33^{rd}, 34^{th} and 35^{th} Olympics are already settled (Paris 2024; Los Angeles 2028; Brisbane 2032), we propose to **collect feedback about joining the TOC from all countries in 2025**.

After professional feasibility verification, we will **help IOC select nine countries** to organize the TOC. Then IOC will plan the construction of Olympic venues as a whole. In 2036, TOC should **begin in Paris**. It will host the "reduced" Summer Olympic Games, according to our event allocation. Fourteen months after that, **Jakarta, Indonesia will host the first Autumn Olympic Games** in 2037. In late 2038 or early 2039, **Innsbruck, Austria** will host the Winter Olympic Games, followed by our **first TOC GAP YEAR in 2039**. The next cycle will begin in 2040.

The original intention of the Olympic Games is to motivate people to stay healthy through sports. In order to let more people around the world participate in Olympic Games, we propose to **avoid the main festivals** in the East and West.

10 Conclusion and Future Work

10.1 Conclusion

To sum up, we first provide the IOC with our suggestion: Triple Olympic Circles, and we briefly describe this recommendation.

Secondly, we establish the Olympic Pattern Evaluation Model to evaluate the costs and benefits of hosting the Olympic Games. Based on this model, we find that the TOC pattern scores 0.64 points while the current Olympic pattern scores only 0.49. It proves that hosting the Olympics by cycle is better than bidding for its sponsorship.

Considering that hosting Summer Olympics is still a burden for most countries, we then divide it into two parts: Summer Olympics and Autumn Olympics. In the meantime, Winter Olympics remains the same as now. Each Olympic event is held by three fixed cities, thus the name "Triple Olympic Circles".

We use TOPSIS and FCM algorithms to find out cities that are suitable to hold one of these Olympics. For Summer Olympics, developed countries have obvious advantages. Tokyo, New York and Paris come to the top 3 in the ranking list. For Autumn Olympics, however, developing countries have more chances to bid. Jakarta, Sao Paulo and Peking win the nominations to be members of the second TOC. For Winter Olympics, only high-latitude countries have basic weather conditions. Among them, Oslo, Stockholm and Innsbruck have clear advantages.

Thereafter, we apply a case study, i.e., predictions of the GDP of Paris, to demonstrate that joining the TOC will increase the GDP growth rate by 1.154%. Since Paris is going to host the 2024 Olympics, it is possible to start our new pattern from 2036, when Paris will host the first reduced Summer Olympics as we suggest.

At last, we discuss the advantages and disadvantages of our model.

10.2 Future Work

In order to maintain a good time interval, we limit the time of the Summer Olympics to August. However, Qatar has already hosted the 2022 World Cup in December. Whether the limit can be removed depends on many political and economic factors and needs to be discussed in detail in the future.

In order to improve our evaluation system, if time permits, we are going to apply the data of each previous hosting bid for the Olympics bid to our model. Then we will verify whether our results are in line with reality and try to promote consistency.

We submit an **Olympic Community policy** to improve the feasibility of our proposal. We suggest host countries **crowdfund** part of the construction fee and provide labor service exchange. Any country participating in crowdfunding will enjoy certain privileges, such as event-relieved broadcasting fees, improved treatment for athletes, quick processing of visitors' visas, advertisement serving priority, etc. The degree of privilege depends on the financial and labor support invested in crowdfunding.

11 Memorandum

Memorandum

To: International Olympic Committee
By: ICGM
Date: 2023.4.3
Subject: Presentation of our proposal – TOC

Nowadays, fewer and fewer countries are showing interest in bidding to host the Olympics, which poses a huge challenge for the future of the Games. To address this issue, our team has come up with the above TOC (Triple Olympic Circles) proposal, which we believe will open up more opportunities and possibilities for the future of the Games.

The core of the proposal is to divide the Olympic Games into three parts, Summer, Autumn, and Winter Olympics. Each of them will be held in three different countries in turn. In this way, different countries can share the responsibility of hosting the Olympic Games, and the cost of it can be reduced. At the same time, by expanding the scope to both developed and developing countries, more countries will have equal opportunities to participate in the Olympics, which is of great significance to the global promotion and popularization of the Olympics.

We propose to adopt a four-year Olympic cycle, in which the Summer, Autumn and Winter Olympic Games will be held in the first three years. In the fourth year, we propose to apply a "GAP YEAR" to prepare for the next cycle. It will allow every country to have sufficient time to prepare for the future Olympics and deal with sudden crises.

Currently, our nine TOC countries face various challenges. Among them, different economic costs when hosting different Olympic Games leads to potential inequity. For example, the cost of the Summer Olympics is higher than that of the Autumn Olympics due to the high cost of venues and facilities. Although benefits are positively correlated with expenditures, risks always make people hesitate. To solve this problem, we propose a “Community” policy.

The policy suggests host countries crowdfund part of construction funds and service support from other countries. All countries participating in crowdfunding will receive preferential treatment in the following areas: event broadcast fee relief, improved treatment of athletes, fast processing of visitor visas, advertisement serving priority… The degree of preferential treatment depends on the financial and labor support invested in crowdfunding. In this policy, countries share the costs and enjoy the benefits of the Olympic Games together. The cooperation between the countries become closer, realizing the dream of forming an Olympic community.

We believe this proposal can effectively solve the problem of fewer and fewer countries bidding to host the Olympic Games. More opportunities and possibilities are brought for the future Olympic Games. We hope you will give this suggestion serious consideration and discuss with us how to make it become reality.

Thank you for your time and patience in reading!

Sincerely,

ICGM
2023.4.3

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